

**Report on the Effectiveness of Mathnasium Learning Center Teaching
on Elementary and Middle School Student
Performance on Standards-based Mathematics Tests**

prepared by

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January, 2004

Summary

In 2003, 35 elementary and middle school students attending the Mathnasium Learning Center participated in a study to determine the effectiveness of the program. On entry into the program, the students were given standards-based placement tests to determine an individual course of action for each student. This placement test served as a pre-test. After an average treatment period of more than 3 months, the students were again assessed, this time with a posttest. Analysis of the 2nd and 5th grade students showed a statistically significant improvement in the Center's math test scores.

Introduction

Mathnasium is a learning center where kids go after school to boost their math skills. The center is highly specialized; teaching only math. The program is for students in grades 2 through 8. Students attend the center once or twice a week, for about an hour. Like a gym or health club, members pay a monthly fee and can drop-in anytime. The goal is to significantly increase a student's math skills, understanding of math concepts, and overall school performance, while building confidence and forging a positive attitude toward the subject.

The company sought to determine the effectiveness of its program, and set in motion several qualitative and quantitative studies.

In early Fall, 2003, after five months of operations, the parents of the Mathnasium Learning Center in Los Angeles were given a survey in order to gauge their feelings about the impact of the program. Two primary questions were asked: "How did your child's grade in math at school change since enrolment at Mathnasium?" And, "How has your child's attitude towards math improved since enrolment at Mathnasium?" The results of this qualitative study were that 67% of parents reported their children's grades improved, 41% of those "significantly"; and 85% of parents said their children's attitude toward math had improved (Mathnasium, 2004).

In addition to the qualitative study, quantitative studies have been considered. The first, this study, is designed to determine the effectiveness of the Learning Center's program in a small scale, single group non-experimental pre-posttest design. This study has been commissioned to determine whether there exists a positive treatment effect on mathematics testing performance of elementary and middle school children as a result of their attending the Mathnasium teaching center for a period of more than 3 months.

Research Method

To see whether the students' skills are improving as a result of Mathnasium teaching, two math tests will be given to each student, one at the beginning of the study period (pretest), and one at the end (posttest). The tests will test ability in similar skills, the skills that the treatment program (Mathnasium Learning Center mathematics program) is supposed to enhance. The pre and posttests will be aligned to California State standards. The tests will be validated by an experienced credentialed mathematics teacher, showing that they really test what they say they test.

Between the two tests, each student will attend the Learning Center approximately once per week for mathematics lessons. Because of the flexible nature of the Learning Center, the treatment period will vary from 3-5 months depending on when students start with the program.

The design of this statistical study is a 'Single Group Pretest-Posttest Design' (Figure 1). This design compares the same group of participants before and after the program. The purpose of the single group pretest-posttest design is to determine if participants improved after receiving the program.

It should be noted that this is a non-experimental design, serving as a pilot and an easier to implement and less expensive study than experimental, or quasi-experimental designs. But, this design has inherent limitations, which will be noted in the Conclusions and Recommendations section of this report. Some of these limitations may be mitigated partially by the timing of this study. Much of the treatment period occurred over the summer months in 2003, when students were not in school, and the treatment program was the only mathematics learning program the students were exposed to.

Figure 1. Single Group Research Design based on Kerlinger (1973)

Students at the Mathnasium Learning Center form a single group. The group receives the treatment for a minimal period of three months. O represents the pretest and posttest.

O X O

Once the data is collected at the end of the study, the data can be input into a statistics program and *t*-test can be run to see if there is any difference in performance on tests.

The null hypothesis of this study is that attending the Learning Center will have no positive causal effect on posttest performance. A *t*-test comparing matched pairs of pre and posttest results will statistically determine if there is a significant difference between the two test scores across the study population.

Analysis

Once the pre and posttest data was collected, the data was reviewed to determine which grades could serve as the subject of further statistical analysis. Paired pre and posttest data was collected for grades 2, 3, 4, 5, and Middle School (MS). The goal was to identify two grades with a reasonably normal distribution of scores, and minimally sufficient sample size to calculate t-tests. Grades 3, MS (low sample sizes) and 4 (non-normal distribution) were not advanced to the statistical portion of the study. Thus, the *t-test* analysis would be performed on the data collected for grades 2 and 5.

The researchers attempted to minimize human input and manipulation of data. This was achieved by the following steps that involved automated computer tools:

1. Use Microsoft Excel to transpose a row per student per test to column format,
2. Export the Excel columns of data to SQL-compatible data structures, and
3. Run an SQL query-based scoring algorithm on the data to calculate final scores (percentage correct) for each test.

Once final percentage scores were calculated, the data from the two grades were imported into SPSS for Windows (version 11), and t-tests were run. The results are can be found in Tables 1 – 6. Tables 1 -3 show the results for the grade 2 data. The paired samples correlate highly (Table 2). A statistically significant difference in the testing scores between pre and posttest is shown at the 95% confidence level.

Table 1. Paired Samples Statistics, Grade 2

	Mean	N	Std. Deviation	Std. Error Mean
PRETESTPC2	45.57	7	21.110	7.979
POSTTESTP2	66.71	7	17.385	6.571

Table 2. Paired Samples Correlations, Grade 2

N	Correlation	Sig.
7	.741	.057

Table 3. Paired Samples Test, Grade 2

Mean	Std. Deviation	Paired Differences			t	Df	Sig. (2-tailed)
		Std. Error Mean	95% Confidence Interval of the Difference				
			Lower	Upper			
-21.14	14.288	5.400	-34.36	-7.93	-3.915	6	.008

Tables 4 -6 show the results for the grade 5 data. The paired samples correlate highly (Table 5). A statistically significant difference in the testing scores between pre and posttest is shown at the 95% confidence level.

Table 4. Paired Samples Statistics, Grade 5

	Mean	N	Std. Deviation	Std. Error Mean
PRETESTPC2	42.67	9	16.039	5.346
POSTTESTP2	52.78	9	17.130	5.710

Table 5. Paired Samples Correlations, Grade 5

N	Correlation	Sig.
9	.697	.037

Table 6. Paired Samples Test, Grade 5

Paired Differences					t	df	Sig. (2-tailed)
Mean	Std. Deviation	Std. Error Mean	95% Confidence Interval of the Difference				
			Lower	Upper			
-10.11	12.956	4.319	-20.07	-.15	-2.341	8	.047

Conclusion and Recommendations

The statistical results show a positive treatment effect. The mean score for the 2nd grade students rose from 46% to 67% while the 5th grade student scores rose from 43% to 53% (all scores rounded to the nearest percent). The students performed significantly better on a math post-test after receiving instruction through the learning center.

While these results show a positive treatment effect, it is recommended that a larger scale, qualitative, experimental study be considered within a controlled environment and time frame. This research is designed to supplement other studies to determine the effectiveness of the Learning Center. This was a non-experimental design, serving as a pilot for the learning center's exploration of its program's effectiveness. As such, this study was easier to implement and less expensive study than experimental, or quasi-experimental designs. But, this design has inherent limitations, namely participants may improve over time without intervention of any kind, and these changes can be mistakenly attributed to the program under evaluation. This limitation may have been mitigated partially by the timing of this study. Much of the treatment period occurred over the summer months in 2003, when school was out of session. This design could not indicate, however, whether the program solely caused improvement in participants; as there is no way to distinguish between changes over time due to other factors and effects specific to the program.

It is recommended that a larger scale, qualitative, experimental study be considered within a controlled environment and time frame. A very sound approach to an experiment would be to have two groups, one which is a 'control', or group that does not receive the treatment, and the other which is 'experimental' or 'treatment', the group which uses the software. The purpose of control is to reduce and bias. Size of sample was very small in this study, and it is recommended that the center conduct additional studies using larger numbers of students. To produce reliable statistics, the minimum size of the groups ought to be a minimum of 20 subjects per group; of course, the larger the group, the better.

Despite the limits encountered in this study, when coupled with qualitative feedback from parents demonstrating that student's positive attitude toward learning math has increased, and their children's grades improved, the results of this study are promising.

References

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Appendix A. Mathnasium Corporate Information



Mathnasium Learning Centers
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Appendix B. Mathnasium Teaching Philosophy, Method, and Curriculum

Philosophy

The key to understanding math is Number Sense. Number Sense does not develop by accident. It is the result of a process of encounter and interaction with a specific set of concepts and skills presented in a way that makes sense to the learner. The Mathnasium Method is the life's work of Larry Martinek, Mathnasium's Chief Education Officer and a teacher and math teaching consultant in the Los Angeles area for the past 30 years. It's the best there is: a time-tested, personalized program, that employs diagnostics, instruction, worksheets, manipulatives, and the latest computer software to build Number Sense, and with it, confidence and a deep understanding and lifelong love of mathematics.

Strategy

Learning from the successes and failures of other approaches, and from the teaching experience of its creator, The Mathnasium Method uses a unique combination of mental, verbal, visual, tactile, and written techniques to help children learn math.

MENTAL

Students are taught how and when to use mental math techniques. This enables them to dispense with needless paper-and-pencil work and focus on the task at hand.

Example: $99 + 99 + 99 = \underline{\quad}$

Instead of setting this problem as a vertical addition problem, students are taught to think, " $100 + 100 + 100 - 3 = 300 - 3 = 297$."

VERBAL

Language is used as an integral part of the program. Students are taught the meaning of root words in the mathematics context. Students are also taught how

to explain their thought process and reasoning verbally.

Example: Percent

Percent is taught as meaning per CENT, “for each 100.” Using this definition, “7% of 300” is easily seen to be, “7 for the first 100, 7 for the second hundred, and 7 for the third hundred = $7 + 7 + 7 = 21$.”

VISUAL

Meaningful pictures, charts, and tables are used to explain ideas and concepts.

Many of the problems in the workbooks are “pictured-based,” providing students with insights into problems that transcend the written words.

Example: If each circle in the picture is a dime, how much money is shown in the picture?

Many of the problems in the Mathnasium program feature pictures as prompts for problem solving.

TACTILE

When appropriate, manipulatives are used to introduce, explain, and/or reinforce concepts and skills.

The transfer of knowledge from manipulatives to other aspects of learning is carefully monitored.

Examples: Counting chips are used to facilitate learning the principles of addition, subtraction, multiplication, and division. Dice and cards are used in studying Probability.

WRITTEN

Written practice with computation (“drill”) is a necessary component of mathematics education. Mathnasium provides for abundant practice.

In addition, our workbooks and other printed material provide a framework for the orderly development of mathematical thought and skills.

Examples: Our worksheets cover the entire spectrum from practicing “ $1 + 1$ ” to solving linear equations. In addition, our printed materials cover all aspects of Problem Solving.

ATTITUDE and SELF-ESTEEM

Many students come through our doors with an “I’m no good at math...I hate math” attitude. Kids don’t really “hate math.” What they hate is being, frustrated, embarrassed, and confused by math.

Being successful is the best way to over-come these problems. Mathnasium

provides for success by finding the right starting point (through diagnostic testing) and building confidence and self-esteem through successful encounter and interaction with carefully selected materials.

IN ADDITION

The Mathnasium Method also provides: enrichment at all levels of the curriculum, advanced work, including topics not usually introduced in the classroom, for students who are ready, and intensive remediation, as needed.

Method

EVALUATE

Mathnasium students are given a two-part diagnostic test. The first is a written test designed to assess the student's weakness with respect to grade-level material. The second part is a series of oral questions, designed to assess the depth of the student's understanding of key math concepts and skills. We use the results to assign a learning plan tailor-made for your child.

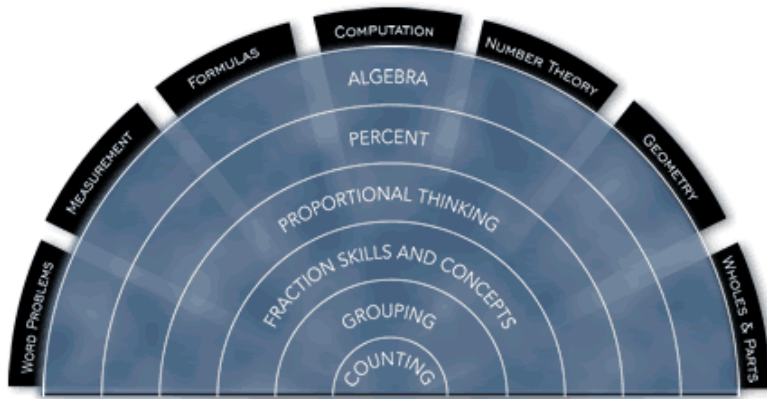
EDUCATE

- ❖ Customized Program for your Child
- ❖ Highly trained instructors
- ❖ Guided practice
- ❖ The latest computer software
- ❖ Manipulatives
- ❖ Periodic assessment to keep students on track
- ❖ Kids workout once or twice a week, or as often as they like, just like a gym.

RESULTS

Your child's progress is measured by his or her grades, third party assessment (ERB, CTBS, ISEE, SAT9/6, CAT), and love of mathematics.

Curriculum



The heart of the Mathnasium curriculum is comprised of:

COUNTING

Counting is "the ability to count from any number, to any number, by any number."

WHOLE & PARTS

Knowledge of Wholes and Parts is "the ability to 'see' wholes and parts in a given question, and to utilize the idea the 'The whole equals the sum of its parts,' and 'Each part equals the whole minus all of the other parts' to answer the question at hand."

PROPORTIONAL THINKING & CHANGE

Proportional Thinking and Change is "the ability to compare numbers by division and by subtraction, and to use this knowledge to solve problems by 'reasoning in groups.'"

These categories are further subdivided into the following 20 curricular areas.

1. Counting
2. Percent
3. Number Facts
4. Measurement
5. Half
6. Geometry
7. Computation
8. Wholes and Parts
9. Proportional Thinking
10. Money
11. SAMEness, Quantity, Value
12. Data Analysis
13. Laws of Mathematics
14. Patterns
15. Negative Numbers
16. Algebraic Thinking

17. Fraction Concepts
18. Problem Solving
19. Number Theory
20. Math Vocabulary

Appendix C. Mathnasium Internal Pre-Tests used in this Study

• Math Checkup #2 •

page 1 of 4

Grade _____

1)
$$\begin{array}{r} 87 \\ + 92 \\ \hline \end{array}$$

2) $83 + 163 + 248 = \underline{\quad}$

3)
$$\begin{array}{r} 73 \\ - 21 \\ \hline \end{array}$$

4)
$$\begin{array}{r} 91 \\ - 34 \\ \hline \end{array}$$

5)
$$\begin{array}{r} 806 \\ - 47 \\ \hline \end{array}$$

6)
$$\begin{array}{r} 200 \\ - 21 \\ \hline \end{array}$$

7) $30 - 11 = \underline{\quad}$

8) $100 - 26 = \underline{\quad}$

9) a) $4 + \underline{\quad} = 12$ b) $8 + \underline{\quad} = 13$

10) a) $19 + \underline{\quad} = 30$ b) $78 + \underline{\quad} = 100$

11) a) $12 - \underline{\quad} = 5$ b) $17 - \underline{\quad} = 9$

12) a) $20 - \underline{\quad} = 8$ b) $20 - \underline{\quad} = 8$

13) Count by **3s**:
0, 3, 6, , , , , , , , , 30, ,

14) Count by **11s**:
0, 11, 22, , , , , , 77, , , , ,

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15) a) 1 foot = _____ inches b) 1 hour = _____ minutes

c) 14 days = _____ weeks d) 3 minutes = _____ seconds

16) a)
$$\begin{array}{r} 5 \\ \times 4 \\ \hline \end{array}$$
 b)
$$\begin{array}{r} 6 \\ \times 3 \\ \hline \end{array}$$
 c)
$$\begin{array}{r} 7 \\ \times 5 \\ \hline \end{array}$$

d) $5 \times \underline{\quad} = 30$ e) $23 \times \underline{\quad} = 0$

17) a) Which digit is in the *one's* place: 80 _____

b) Which digit is in the *ten's* place: 369? _____

18) a) What number is 10 more than 35? _____

b) What number is 20 less than 75? _____

19) a) Five dimes is the same amount as how many nickels? _____
(5 dimes = _____ nickels)

b) Ten quarters is the same amount as how many dimes? _____
(10 quarters = _____ dimes)

20) a) $39 + 22 = \underline{\quad}$ b) $49 + \underline{\quad} = 101$

- 21) On Monday, Marie picked 5 flowers. On Tuesday, she picked 4 flowers. How many did she pick altogether?

- 22) A boy had 12 marbles. He gave away 3 to a friend. How many marbles did he have left?

- 23) A 20-foot piece of wood is cut into three pieces. One piece is 3 feet long and the second piece is 4 feet long. How long is the third piece?

- 24) Half-a-pound of candy costs 75 cents. How much does a whole pound of candy cost?

- 25) Jan bought two pencils that cost 10 cents each. She also bought 3 erasers the cost 5 cents each. How much money did she spend altogether?

- 26) If you buy something for 35 cents, how much change will you get from a dollar?

27) Recyclers pay 5 cents for every 2 cans returned.

a) How much are 6 cans worth? _____

b) How many cans are needed to get 25 cents? _____

28) a) half of 20 = _____

b) half of 42 = _____

c) half of 7 = _____

d) half of 21 = _____

e) half of 99 = _____

f) half of $6\frac{1}{2}$ = _____

g) half of _____ = 8

h) half of _____ = $4\frac{1}{2}$

i) half of _____ = $24\frac{1}{2}$

j) half of _____ = $5\frac{1}{4}$

29) Name six numbers that are between 4 and 7: _____

30) Continue the pattern:

a) **0 2 4 6 8 10 12** _____

b) **OX OOX OOOX** _____

c) **0 10 9 19 18 28 27** _____

• Math Checkup #5 •

Show your work, neatly.

Grade

Date

Name

1) a)
$$\begin{array}{r} 3,482 \\ - 724 \\ \hline \end{array}$$
 b)
$$\begin{array}{r} 73,000 \\ - 9,123 \\ \hline \end{array}$$
 c)
$$\begin{array}{r} 67,008 \\ - 2,345 \\ \hline \end{array}$$

2) a) $0.8 + 24.123 = \underline{\hspace{2cm}}$ b) $4.25 + 3 = \underline{\hspace{2cm}}$

3) a) $8.28 - 0.3 = \underline{\hspace{2cm}}$ b) $8 - 2.75 = \underline{\hspace{2cm}}$

4) a)
$$\begin{array}{r} 31.2 \\ \times 4 \\ \hline \end{array}$$
 b)
$$\begin{array}{r} 6.67 \\ \times 0.03 \\ \hline \end{array}$$
 c)
$$\begin{array}{r} 0.4 \\ \times 0.2 \\ \hline \end{array}$$

5) a) $0.7 \overline{) 1.4}$ b) $1.6 \overline{) 2.56}$ c) $0.2 \overline{) 73}$

6) a)
$$\begin{array}{r} \frac{3}{4} \\ + \frac{3}{4} \\ \hline \end{array}$$
 b)
$$\begin{array}{r} \frac{3}{8} \\ + \frac{5}{6} \\ \hline \end{array}$$
 c)
$$\begin{array}{r} 2\frac{3}{4} \\ + 4\frac{2}{3} \\ \hline \end{array}$$

7) a)
$$\begin{array}{r} \frac{7}{8} \\ - \frac{3}{8} \\ \hline \end{array}$$
 b)
$$\begin{array}{r} 6 \\ - 3\frac{3}{5} \\ \hline \end{array}$$
 c)
$$\begin{array}{r} 6\frac{1}{3} \\ - 2\frac{3}{4} \\ \hline \end{array}$$

8) a)
$$\begin{array}{r} 321 \\ \times 12 \\ \hline \end{array}$$
 b)
$$\begin{array}{r} 708 \\ \times 97 \\ \hline \end{array}$$
 c)
$$\begin{array}{r} 321 \\ \times 123 \\ \hline \end{array}$$

9) a)
$$\begin{array}{r} \\ 8) 8,169 \\ \hline \end{array}$$
 b)
$$\begin{array}{r} \\ 16) 2,560 \\ \hline \end{array}$$
 c)
$$\begin{array}{r} \\ 67) 6,245 \\ \hline \end{array}$$

10) a) $\frac{8}{9} \times \frac{3}{4} = \underline{\hspace{2cm}}$ b) $\frac{4}{5} \times \frac{1}{2} = \underline{\hspace{2cm}}$

11) a) $\frac{8}{9} + \frac{4}{5} = \underline{\hspace{2cm}}$ b) $\frac{4}{9} \div \frac{2}{3} = \underline{\hspace{2cm}}$

12) a) Circle the fraction with the *greatest* value: $\frac{12}{13}$, $\frac{19}{20}$, $\frac{3}{7}$, $\frac{77}{99}$

b) Arrange in order from *smallest* to *largest*: 0, 1, $\frac{1}{2}$, $\frac{5}{12}$, $\frac{5}{8}$

13) a) $\frac{3}{4}$ of 20 = _____ b) $\frac{5}{8}$ of 24 = _____

c) $\frac{1}{5}$ of _____ = 7 d) $\frac{3}{5}$ of _____ = 9

14) Draw a rectangle. Label the length $12\frac{1}{2}$ feet and the width 2 feet.

a) Find the PERIMETER of the rectangle. _____

b) Find the AREA of the rectangle. _____

15) a) Write in words: 87,654,321 _____

b) Write in words: 5.234 _____

16) a) 25% of 20 = _____

b) 8% of 300 = _____

c) 16% of 225 = _____

d) 12% of 130 = _____

17) 10% of the apples in a barrel were rotten. The barrel contained a total of 120 apples. How many apples were *not* rotten? _____

18) A bag contains 5 red M&Ms, 4 green M&Ms, and 3 blue M&Ms.

a) What is the ratio of blue M&Ms to green M&Ms? _____

b) What is the ratio of green M&Ms to red M&Ms? _____

c) What part of the M&Ms are red? _____

- 19) 3,000 pounds of sand is used to fill sandboxes. 1,000 pounds was used to fill the big sandbox, and the rest was used to fill four equal-size sandboxes. How much sand was put in each of the smaller sandboxes?

- 20) Martha and George take turns baby-sitting. Martha works for 4 hours and George works for 2 hours. They are paid \$24.00.

How should they share the money so that each person gets a fair share?

Martha = _____ George = _____

- 21) a) $7 + -3 =$ _____ b) $-9 + 7 =$ _____
c) $-8 + -7 =$ _____ d) $-1 + 2 + -7 + -2 =$ _____
e) $4 - 7 =$ _____ f) $-3 - -7 =$ _____
g) $-5 + \underline{\quad} = -2$ h) $-4 - \underline{\quad} = 9$
- 22) a) Round to nearest *thousand*: 1,237,421.5: _____
b) Round to nearest *hundredth*: 24.3472: _____
c) Round to nearest *tenth*: 129.9521: _____
- 23) a) What is the *sum* of 10 and 2? _____
b) What is the *product* of 10 and 2? _____
c) What is the *quotient* of 10 and 2? _____
d) What is the *difference* of 10 and 2? _____

24) Define:

- a) DENOMINATOR _____
- b) MULTIPLE _____
- c) FACTOR _____

25) What is the next number in the pattern:

- a) 0, 1, 3, 6, 10, 15, _____ b) 0, 1, 1, 2, 3, 5, 8, 13, _____

26) List *all* the factors of:

- a) 12 _____ b) 60 _____

27) What is the GCF (Greatest Common Factor) of:

- a) 18 and 24 GCF = _____ b) 75 and 125 GCF = _____

28) What is the LCM (Least Common Multiple) of:

- a) 6 and 8 LCM = _____ b) 10 and 12 LCM = _____

29) A bag contains 10 red marbles, 8 white marbles, and 6 blue marbles. If one marble is picked from the bag, what is the probability that the marble will be:

- a) a blue marble _____ b) *not* a white marble _____?

- 30) A furlong is one-eighth ($\frac{1}{8}$) of a mile. What part of a mile is 6 furlongs? _____
- 31) Papa bear ate half of the pie. Mama bear ate half of what was left. Baby bear ate half of what was left.
- a) What part of the pie was left for Goldilocks? _____
- b) What part of the pie did Mama bear get? _____
- 32) Name five numbers between $\frac{1}{4}$ and $\frac{3}{4}$: _____

Appendix D. Test Alignments to California State Standards

Grade 2 Test PT2

ItemID	CA Strand	CA Standard Description
1	Number Sense	2.2 Find the sum or difference of two whole numbers up to three digits long.
2	Number Sense	2.2 Find the sum or difference of two whole numbers up to three digits long.
3	Number Sense	2.2 Find the sum or difference of two whole numbers up to three digits long.
4	Number Sense	2.2 Find the sum or difference of two whole numbers up to three digits long.
5	Number Sense	2.2 Find the sum or difference of two whole numbers up to three digits long.
6	Number Sense	2.2 Find the sum or difference of two whole numbers up to three digits long.
7	Number Sense	2.2 Find the sum or difference of two whole numbers up to three digits long.
8	Number Sense	2.2 Find the sum or difference of two whole numbers up to three digits long.
9a	Algebra and Functions	1.1 Use the commutative and associative rules to simplify mental calculations and to check results.
9b	Algebra and Functions	1.1 Use the commutative and associative rules to simplify mental calculations and to check results.
10a	Algebra and Functions	1.1 Use the commutative and associative rules to simplify mental calculations and to check results.
10b	Algebra and Functions	1.1 Use the commutative and associative rules to simplify mental calculations and to check results.
11a	Algebra and Functions	1.1 Use the commutative and associative rules to simplify mental calculations and to check results.
11b	Algebra and Functions	1.1 Use the commutative and associative rules to simplify mental calculations and to check results.
12a	Algebra and Functions	1.1 Use the commutative and associative rules to simplify mental calculations and to check results.
12b	Algebra and Functions	1.1 Use the commutative and associative rules to simplify mental calculations and to check results.
13	Number Sense	3.1 Use repeated addition, arrays, and counting by multiples to do multiplication.
14	Number Sense	3.1 Use repeated addition, arrays, and counting by multiples to do multiplication.
15a	Measurement and Geometry	1.4 Tell time to the nearest quarter hour and know relationships of time (e.g., minutes in an hour, days in a month, weeks in a year).
15b	Measurement and Geometry	1.4 Tell time to the nearest quarter hour and know relationships of time (e.g., minutes in an hour, days in a month, weeks in a year).
15c	Measurement and Geometry	1.4 Tell time to the nearest quarter hour and know relationships of time (e.g., minutes in an hour, days in a month, weeks in a year).
15d	Measurement and Geometry	1.4 Tell time to the nearest quarter hour and know relationships of time (e.g., minutes in an hour, days in a month, weeks in a year).
16a	Number Sense	3.3 Know the multiplication tables of 2s, 5s, and 10s (to "times 10") and commit them to memory.
16b	Number Sense	3.3 Know the multiplication tables of 2s, 5s, and 10s (to "times 10") and commit them to memory.
16c	Number Sense	3.3 Know the multiplication tables of 2s, 5s, and 10s (to "times 10") and commit them to memory.
16d	Number Sense	3.3 Know the multiplication tables of 2s, 5s, and 10s (to "times 10") and commit them to memory.
16e	Number Sense	3.3 Know the multiplication tables of 2s, 5s, and 10s (to "times 10") and commit them to memory.
17a	Number Sense	1.1 Count, read, and write whole numbers to 1,000 and identify the place value for each digit.
17b	Number Sense	1.1 Count, read, and write whole numbers to 1,000 and identify the place value for each digit.
19a	Number Sense	5.0 Students model and solve problems by representing, adding, and subtracting amounts of money

19b	Number Sense	5.0 Students model and solve problems by representing, adding, and subtracting amounts of money
20a	Algebra and Functions	1.3 Solve addition and subtraction problems by using data from simple charts, picture graphs, and number sentences.
20b	Algebra and Functions	1.3 Solve addition and subtraction problems by using data from simple charts, picture graphs, and number sentences.
21	Algebra and Functions	1.2 Relate problem situations to number sentences involving addition and subtraction.
22	Algebra and Functions	1.2 Relate problem situations to number sentences involving addition and subtraction.
23	Algebra and Functions	1.3 Solve addition and subtraction problems by using data from simple charts, picture graphs, and number sentences.
24	Algebra and Functions	1.3 Solve addition and subtraction problems by using data from simple charts, picture graphs, and number sentences.
25	Mathematical Reasoning	1.1 Determine the approach, materials, and strategies to be used to set up a problem.
26	Number Sense	5.0 Students model and solve problems by representing, adding, and subtracting amounts of money
30a	Statistics, Data Analysis, and Probability	2.1 Recognize, describe, and extend patterns and determine a next term in linear patterns (e.g., 4, 8, 12 ...; the number of ears on one horse, two horses, three horses, four horses).
30b	Statistics, Data Analysis, and Probability	2.1 Recognize, describe, and extend patterns and determine a next term in linear patterns (e.g., 4, 8, 12 ...; the number of ears on one horse, two horses, three horses, four horses).
30c	Statistics, Data Analysis, and Probability	2.1 Recognize, describe, and extend patterns and determine a next term in linear patterns (e.g., 4, 8, 12 ...; the number of ears on one horse, two horses, three horses, four horses).

Grade 5, Test PT5A

2a	Number Sense	2.1 Add, subtract, multiply, and divide with decimals; add with negative integers; subtract positive integers from negative integers; and verify the reasonableness of the results.
2b	Number Sense	2.1 Add, subtract, multiply, and divide with decimals; add with negative integers; subtract positive integers from negative integers; and verify the reasonableness of the results.
3a	Number Sense	2.1 Add, subtract, multiply, and divide with decimals; add with negative integers; subtract positive integers from negative integers; and verify the reasonableness of the results.
3b	Number Sense	2.1 Add, subtract, multiply, and divide with decimals; add with negative integers; subtract positive integers from negative integers; and verify the reasonableness of the results.
—	Number Sense	2.1 Add, subtract, multiply, and divide with decimals; add with negative integers; subtract positive integers from negative integers; and verify the reasonableness of the results.
4a	Number Sense	2.1 Add, subtract, multiply, and divide with decimals; add with negative integers; subtract positive integers from negative integers; and verify the reasonableness of the results.
4b	Number Sense	2.1 Add, subtract, multiply, and divide with decimals; add with negative integers; subtract positive integers from negative integers; and verify the reasonableness of the results.
4c	Number Sense	2.1 Add, subtract, multiply, and divide with decimals; add with negative integers; subtract positive integers from negative integers; and verify the reasonableness of the results.
5a	Number Sense	2.2 Demonstrate proficiency with division, including division with positive decimals and long division with multidigit divisors.
5b	Number Sense	2.2 Demonstrate proficiency with division, including division with positive decimals and long division with multidigit divisors.
5c	Number Sense	2.2 Demonstrate proficiency with division, including division with positive decimals and long division with multidigit divisors.
6a	Number Sense	2.3 Solve simple problems, including ones arising in concrete situations, involving the addition and subtraction of fractions and mixed numbers (like and unlike denominators of 20 or less), and express answers in the simplest form.
6b	Number Sense	2.3 Solve simple problems, including ones arising in concrete situations, involving the addition and subtraction of fractions and mixed numbers (like and unlike denominators of 20 or less), and express answers in the simplest form.
6c	Number Sense	2.3 Solve simple problems, including ones arising in concrete situations, involving the addition and subtraction of fractions and mixed numbers (like and unlike denominators of 20 or less), and express answers in the simplest form.
7a	Number Sense	2.3 Solve simple problems, including ones arising in concrete situations, involving the addition and subtraction of fractions and mixed numbers (like and unlike denominators of 20 or less), and express answers in the simplest form.
7b	Number Sense	2.3 Solve simple problems, including ones arising in concrete situations, involving the addition and subtraction of fractions and mixed numbers (like and unlike denominators of 20 or less), and express answers in the simplest form.
7c	Number Sense	2.3 Solve simple problems, including ones arising in concrete situations, involving the addition and subtraction of fractions and mixed numbers (like and unlike denominators of 20 or less), and express answers in the simplest form.
9a	Number Sense	2.2 Demonstrate proficiency with division, including division with positive decimals and long division with multidigit divisors.
9b	Number Sense	2.2 Demonstrate proficiency with division, including division with positive decimals and long division with multidigit divisors.
9c	Number Sense	2.2 Demonstrate proficiency with division, including division with positive decimals and long division with multidigit divisors.
10a	Number Sense	2.5 Compute and perform simple multiplication and division of fractions and apply these procedures to solving problems.
10b	Number Sense	2.5 Compute and perform simple multiplication and division of fractions and apply these procedures to solving problems.
11a	Number Sense	2.5 Compute and perform simple multiplication and division of fractions and apply these procedures to solving problems.
11b	Number Sense	2.5 Compute and perform simple multiplication and division of fractions and apply these procedures to solving problems.
14a	Measurement and Geometry	1.1 Derive and use the formula for the area of a triangle and of a parallelogram by comparing it with the formula for the area of a rectangle (i.e., two of the same triangles make a parallelogram with twice the area; a parallelogram is compared with a rectangle of the same area by cutting and pasting a right triangle on the parallelogram).
14b	Measurement and Geometry	1.1 Derive and use the formula for the area of a triangle and of a parallelogram by comparing it with the formula for the area of a rectangle (i.e., two of the same triangles make a parallelogram with twice the area; a parallelogram is compared with a rectangle of the same area by cutting and pasting a right triangle on the parallelogram).
15a	Number Sense	1.1 Estimate, round, and manipulate very large (e.g., millions) and very small (e.g., thousandths) numbers.
15b	Number Sense	1.1 Estimate, round, and manipulate very large (e.g., millions) and very small (e.g., thousandths) numbers.
16a	Number Sense	2.5 Compute and perform simple multiplication and division of fractions and apply these procedures to solving problems.
16b	Number Sense	2.5 Compute and perform simple multiplication and division of fractions and apply these procedures to solving problems.
16c	Number Sense	2.5 Compute and perform simple multiplication and division of fractions and apply these procedures to solving problems.
16d	Number Sense	2.5 Compute and perform simple multiplication and division of fractions and apply these procedures to solving problems.

18a	Statistics, Data Analysis, and Probability	1.3 Use fractions and percentages to compare data sets of different sizes.
18b	Statistics, Data Analysis, and Probability	1.3 Use fractions and percentages to compare data sets of different sizes.
18c	Statistics, Data Analysis, and Probability	1.3 Use fractions and percentages to compare data sets of different sizes.
21a	Number Sense	2.1 Add, subtract, multiply, and divide with decimals; add with negative integers; subtract positive integers from negative integers; and verify the reasonableness of the results.
21b	Number Sense	2.1 Add, subtract, multiply, and divide with decimals; add with negative integers; subtract positive integers from negative integers; and verify the reasonableness of the results.
21c	Number Sense	2.1 Add, subtract, multiply, and divide with decimals; add with negative integers; subtract positive integers from negative integers; and verify the reasonableness of the results.
21d	Number Sense	2.1 Add, subtract, multiply, and divide with decimals; add with negative integers; subtract positive integers from negative integers; and verify the reasonableness of the results.
21e	Number Sense	2.1 Add, subtract, multiply, and divide with decimals; add with negative integers; subtract positive integers from negative integers; and verify the reasonableness of the results.
21f	Number Sense	2.1 Add, subtract, multiply, and divide with decimals; add with negative integers; subtract positive integers from negative integers; and verify the reasonableness of the results.
21g	Number Sense	2.1 Add, subtract, multiply, and divide with decimals; add with negative integers; subtract positive integers from negative integers; and verify the reasonableness of the results.
21h	Number Sense	2.1 Add, subtract, multiply, and divide with decimals; add with negative integers; subtract positive integers from negative integers; and verify the reasonableness of the results.
22a	Number Sense	1.1 Estimate, round, and manipulate very large (e.g., millions) and very small (e.g., thousandths) numbers.
22b	Number Sense	1.1 Estimate, round, and manipulate very large (e.g., millions) and very small (e.g., thousandths) numbers.
22c	Number Sense	1.1 Estimate, round, and manipulate very large (e.g., millions) and very small (e.g., thousandths) numbers.
25a	Mathematical Reasoning	1.1 Analyze problems by identifying relationships, distinguishing relevant from irrelevant information, sequencing and prioritizing information, and observing patterns.
25b	Mathematical Reasoning	1.1 Analyze problems by identifying relationships, distinguishing relevant from irrelevant information, sequencing and prioritizing information, and observing patterns.
26a	Number Sense	2.3 Solve simple problems, including ones arising in concrete situations, involving the addition and subtraction of fractions and mixed numbers (like and unlike denominators of 20 or less), and express answers in the simplest form.
26b	Number Sense	2.3 Solve simple problems, including ones arising in concrete situations, involving the addition and subtraction of fractions and mixed numbers (like and unlike denominators of 20 or less), and express answers in the simplest form.
27a	Number Sense	2.3 Solve simple problems, including ones arising in concrete situations, involving the addition and subtraction of fractions and mixed numbers (like and unlike denominators of 20 or less), and express answers in the simplest form.
27b	Number Sense	2.3 Solve simple problems, including ones arising in concrete situations, involving the addition and subtraction of fractions and mixed numbers (like and unlike denominators of 20 or less), and express answers in the simplest form.
28a	Number Sense	2.3 Solve simple problems, including ones arising in concrete situations, involving the addition and subtraction of fractions and mixed numbers (like and unlike denominators of 20 or less), and express answers in the simplest form.
28b	Number Sense	2.3 Solve simple problems, including ones arising in concrete situations, involving the addition and subtraction of fractions and mixed numbers (like and unlike denominators of 20 or less), and express answers in the simplest form.
29a	Number Sense	2.3 Solve simple problems, including ones arising in concrete situations, involving the addition and subtraction of fractions and mixed numbers (like and unlike denominators of 20 or less), and express answers in the simplest form.
29b	Statistics, Data Analysis, and Probability	1.3 Use fractions and percentages to compare data sets of different sizes.
30	Statistics, Data Analysis, and Probability	1.3 Use fractions and percentages to compare data sets of different sizes.

Appendix E. Raw Data Example

7/22/2003 *	
Grade	1
ID#	1
Item #	N - W - R
1	R
2	R
3	W
4	R
5	R
6	R
7	R
8	R
9	R
10	R
11	R
12	R
13	R
14	R
15	R
16	R
17	R
18	R
19	R
20	R
21	R
22	W
23	R
24	R
25	R
26a	R
26b	R
26c	R
26d	R
27a	W
27b	W
28a	R
28b	R
29a	R
29b	R
29c	R
29c	-
30a	R
30b	R
31a	W
31b	W

Appendix F. SQL Application to Calculate Test Results

```
close data
sele 0
use results
sele 0
use analysis
go top
do while .not. eof()
    select count(studentid) as totitems from results where testid=analysis.tid and
studentid =analysis.uid into cursor temp
    replace analysis.totitems with temp.totitems
    select count(studentid) as totcorr from results where testid=analysis.tid and
studentid =analysis.uid and upper(answer) = "R" into cursor temp
    replace analysis.correct with temp.totcorr
    select count(studentid) as totwrong from results where testid=analysis.tid and
studentid =analysis.uid and upper(answer) = "W" into cursor temp
    replace analysis.wrong with temp.totwrong
    select count(studentid) as totnotans from results where testid=analysis.tid and
studentid =analysis.uid and (upper(answer) = "N" or empty(answer)) into cursor temp
    replace analysis.notanswd with temp.totnotans
    select analysis
    skip
enddo

* Calculate; not scoring not answered questions
replace all pctcorrect with ( correct / (totitems - notanswd) ) * 100 for correct + wrong
> 0
replace all pctcorrect with 0 for correct + wrong = 0

* Calculate; scoring all questions - This Calculation Used for PCTCorrect in Statistics
replace all pctcorrec2 with ( correct / totitems ) * 100 for correct + wrong > 0
replace all pctcorrec2 with 0 for correct + wrong = 0

select 0
use student
zap

select analysis
go top
do while .not. eof()
    select student
    locate for uid = analysis.uid
    if .not. found()
        append blank
        replace uid with analysis.uid
    endif
    if "PT" $ upper( analysis.tid )
        replace student.pretest with analysis.tid
        replace student.pretestpct with analysis.pctcorrect
        replace student.pretestpc2 with analysis.pctcorrec2
    else
        replace student.posttest with analysis.tid
        replace student.posttestpc with analysis.pctcorrect
        replace student.posttestpc2 with analysis.pctcorrec2
    endif
    select analysis
    skip
enddo

select * from student where posttestpc > 0 order by pretest, uid
```

Appendix G. Test Result Data

uid	pretest	pretestpc2	posttest	posttestp2
3	PT2	19.3	PS2	57.89
5	PT2	50.88	PS2	91.23
6	PT2	71.93	PS2	85.96
7	PT2	73.68	PS2	73.68
8	PT2	40.35	PS2	59.65
10	PT2	29.82	PS2	43.86
73	PT2	33.33	PS2	54.39
12	PT3	73.68	PS3	84.21
16	PT3	73.68	PS3	98.25
17	PT3	82.46	PS3	96.49
18	PT3	70.18	PS3	66.67
19	PT3	59.65	PS3	63.16
24	PT4	58.54	PS4	96.34
26	PT4	57.32	PS4	86.59
27	PT4	42.68	PS4	56.1
29	PT4	62.2	PS4	35.37
30	PT4	48.78	PS4	65.85
31	PT4	75.61	PS4	73.17
32	PT4	36.59	PS4	59.76
33	PT4	42.68	PS4	63.41
35	PT4	68.29	PS4	70.73
37	PT5A	25.58	PS5A	47.67
39	PT5A	73.26	PS5A	79.07
43	PT5A	32.56	PS5A	29.07
44	PT5A	56.98	PS5A	69.77
47	PT5A	58.14	PS5A	55.81
49	PT5A	33.72	PS5A	26.74
42	PT5B	29.07	PS5B	59.3
46	PT5B	37.21	PS5B	47.67
48	PT5B	37.21	PS5B	59.3
51	PTMS	58.12	PS6A	70.94
52	PTMS	72.65	PS6A	56.41
55	PTMS	81.2	PS6A	62.39
59	PTMS	29.91	PS6A	35.04
60	PTMS	73.5	PS6A	70.09